ON THE CONVECTIONAL INSTABILITY OF A TWO-COMPONENT MIXTURE IN A GRAVITY FIELD

(O KONVEKTIVNOI NEUSTOICHIVOSTI DVUKHKOMPONENTNOI SMESI V POLE TIAZHESTI)

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The conditions for the appearance of convection in a pure medium which is heated from below have been studied quite extensively. A great number of theoretical and experimental papers are devoted to this problem. The convective stability of a mixture with inhomogeneous temperature and concentration distribution, on the other hand, has not been investigated sufficiently, although this problem in all its aspects is no less interesting. The authors are aware of two theoretical papers devoted to the convective stability of a mixture. Wertheim [1] has obtained small perturbation equations and investigated the stability problem for a mixture in a circular vertical cylinder; in that paper the conditions which give rise to steady convection have been determined (the corresponding problem for a pure medium has been solved by Ostroumov [2]; oscillating disturbances were not considered. It has been shown by Gerasimova* that with certain given boundary conditions, vibrational disturbances are absent provided the characteristics of the mixture satisfy certain inequalities. In this case non-equilibrium is defined only by the monotonic disturbances and the investigation of the stability may be carried out by the use of the variational method suggested by Sorokin [3]. The problem of vibrational stability remained essentially unsolved.

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* Gerasimova, S.B., On the theory of convective phenomena in binary mixtures. Candidate dissertation, University of Perm', 1955.

In this paper a solution is presented of the stability problem of a plane vertical layer of a mixture in a gravity field. Because of the simplicity of the field of flow, an exact solution may be established for the non-steady small perturbation equations. The investigation of the spectrum of the disturbances, in particular, leads to the conclusion that in a mixture in distinction to the case of a pure medium [3], there are two possible forms of instability, which correspond to monotonic and vibrational disturbances. The qualitative peculiarities of the established spectrum of instability evidently are not related to the geometry of the region. We shall note that under certain conditions there occurs a breakdown of equilibrium as the density gradient is directed downward (i.e. heavier fluid toward the bottom). This effect, as will be shown, is explained by the opposing effects of diffusion and of heat conductivity which are peculiar to the mixture.

1. Let us investigate a two-component mixture, of density

$$\rho = \rho_0 (1 - \beta_1 T - \beta_2 C) \tag{1.1}$$

where T and C are the temperature of the mixture and the concentration of the lighter component respectively, measured from a given reference level. If the temperature and the concentration of the mixture are inhomogeneous, then, in general, motion arises. The equations of convection of the mixture with consideration of the effects of thermal diffusion and diffusive thermal conductivity were obtained and discussed by Shaposhnikov [4]

$$\begin{gathered}\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2\mathbf{v} + g(\beta_1 T + \beta_2 C)\boldsymbol{\gamma} \\ \frac{\partial T}{\partial t} + \mathbf{v}\nabla T = (\chi + N\lambda^2 D)\nabla^2 T + N\lambda D\nabla^2 C \\ \frac{\partial C}{\partial t} + \mathbf{v}\nabla C = \lambda D\nabla^2 T + D\nabla^2 C, \qquad \operatorname{div}\mathbf{v} = 0\end{gathered} \tag{1.2}$$

where $\mathbf{v}$ is velocity; p is the pressure, measured from the hydrostatic zero at $T = 0$ and $C = 0$; ρ_0 is the density, corresponding to $T = 0$ and $C = 0$; γ is a unit vector, pointing vertically upward; χ, D and λ are the coefficients of heat conduction, diffusion and of thermal diffusion respectively; N is the thermodynamic coefficient which defines (together with λ) the effect of diffusive thermal conductivity [4]. The parameters of the medium in all the formulas are considered to be constant. We shall also introduce the expressions for the molecular heat flux $\mathbf{q}$ and the diffusive flux of the light component $\mathbf{j}$

$$\mathbf{q} = -\rho_0 c_p[(\chi + N\lambda^2 D)\nabla T + N\lambda D\nabla C], \quad \mathbf{j} = -D[\lambda\nabla T + \nabla C] \tag{1.3}$$

From equation (1.2) it is easily seen that for static equilibrium for which $\mathbf{v} = 0$ and all quantities are independent of time, the temperature T_0 and the concentration C_0 satisfy the equations

$$\nabla^2 T_0 = 0, \qquad \nabla^2 C_0 = 0, \qquad (\beta_1 \nabla T_0 + \beta_2 \nabla C_0) \times \gamma = 0 \qquad (1.4)$$

Hence it follows that the gradient of the density of the mixture in equilibrium is constant and vertical.

Let us investigate the stability of the equilibrium of a plane layer of a mixture, bounded by infinite vertical parallel planes $x = \pm d$. We shall consider the equilibrium when not only the gradient of the density, but also the gradients of the temperature and of the concentration are constant and vertical

$$\nabla T_0 = -A_0 \gamma, \qquad \nabla C_0 = -B_0 \gamma \qquad (1.5)$$

From (1.4) it is seen that the conditions of equilibrium can exist also for more complicated distributions of temperature and concentration. From equation (1.5) it follows that for equilibrium the heat flux $\mathbf{q}$ and the diffusive flux $\mathbf{j}$ are also vertical. Because of the conditions on the lower and upper ends of the layer these fluxes may be chosen independent of each other. Therefore, we shall assume also that the constants A_0 and B_0, which determine the fluxes, are chosen independently. If according to the conditions of the equilibrium problem $\mathbf{j} = 0$ and $\mathbf{q} = 0$, then A_0 and B_0 are found to be related. This was shown in the papers by Wertheim and Gerasimova who investigated such cases.

2. Consider a disturbance of the equilibrium (1.5) of the following form:

$$v_x = 0, \quad v_y = 0, \quad v_z = v\,(x, t), \quad T = T\,(x, t), \quad C = C\,(x, t) \qquad (2.1)$$

(the z-axis is directed vertically upward). Assuming that the pressure gradient is equal to zero (free convection) and assuming that all quantities depend on time according to the law $e^{\sigma t}$, we obtain from (1.2) with the help of (1.5) the perturbation equations

$$\begin{aligned} \sigma v &= \nu v'' + g\,(\beta_1 T + \beta_2 C) \\ \sigma T - A_0 v &= (\chi + N\lambda^2 D)\, T'' + N\lambda DC'' \\ \sigma C - B_0 v &= \lambda DT'' + DC'' \end{aligned} \qquad (2.2)$$

(primes mean differentiation with respect to x). Let us choose the units of distance $l = d/\pi$, of time l^2/ν, of velocity χ/l, of temperature A_0/l and of concentration $B_0 l\chi/D$.

Keeping the previous notation for the variables, equations (2.2) may be represented in non-dimensional form

$$
\begin{aligned}
&\sigma v = v'' + RT + R_d C && \left(R = \frac{g\beta_1 A_0 l^4}{\nu\chi},\ R_d = \frac{g\beta_2 B_0 l^4}{\nu D}\right) \\
&\sigma PT - v = (1 + a)\, T'' + \frac{a}{b} C'' && \left(P = \frac{\nu}{\chi},\ P_d = \frac{\nu}{D}\right) \qquad (2.3) \\
&\sigma P_d C - v = bT'' + C'' && \left(a = \frac{N\lambda^2 D}{\chi},\ b = \frac{\lambda A_0 D}{B_0 \chi}\right)
\end{aligned}
$$

where R and R_d are the usual and the diffusive Rayleigh numbers respectively; P and P_d are the Prandtl numbers; and a and b are nondimensional parameters which characterize the thermal diffusion and diffusive heat conductivity. Equations (2.3) have simple solutions, satisfying the condition of a closed flow

$$\int_{-\pi}^{\pi} v\, dx = 0$$

with the following boundary conditions:

1) At the boundaries of the layer $x \pm \pi$ the velocity and the disturbances of temperature and concentration vanish. In that case v, T and C are proportional to $\sin nx$, where $n = 1, 2, 3, \ldots$

2) At the boundaries of the layer $v' = T' = C' = 0$ (free heat insulated impermeable boundaries). Then v, T and C are proportional to $\sin [2n + 1)x/2]$, where $n = 0, 1, 2, \ldots$

The results in regard to stability obtained from the perturbation investigation of both types are qualitatively the same. Therefore, to be brief, we shall consider here only the first case taking $n = 1$ as the basic level of instability). When substituting the solution of the form (v, T and C) $\sim \sin x$ into the equations (2.3), we obtain a system of linear homogeneous equations for the amplitudes of the disturbances. Upon equating the determinant of this system to zero, we find the equation for the characteristic decrements σ. The sign of the real part of σ indicates the attenuation ($\mathrm{Re}\,\sigma < 0$) or the amplification ($\mathrm{Re}\,\sigma > 0$) of the disturbances; the imaginary part of σ defines the frequency of the disturbances.

The characteristic equation has the form

$$A\sigma^3 + B\sigma^2 + C\sigma + D = 0 \qquad (2.4)$$

where

$$A = PP_d, \qquad B = P + PP_d + (1 + a)\, P_d$$
$$C = 1 + P + (1 + a)\, P_d - P_d R - PR_d$$

$$D = 1 - (1+\alpha)R - (1 + a + a/\alpha)R_d \qquad (\alpha = -\lambda\beta_2/\beta_1)$$

The real and imaginary parts of the characteristic decrement $\sigma = \sigma_1 + i\sigma_2$ satisfy the equations

$$A(\sigma_1^3 - 3\sigma_1\sigma_2^2) + B(\sigma_1^2 - \sigma_2^2) + C\sigma_1 + D = 0 \qquad (2.5)$$

$$\sigma_2[A(3\sigma_1^2 - \sigma_2^2) + 2\sigma_1 B + C] = 0 \qquad (2.6)$$

It is seen from these equations that there exist two types of disturbances:

1) monotonic time varying disturbances $\sigma_2 = 0$:

2) periodic disturbances, $\sigma_2 \neq 0$.

First consider the monotonic disturbances. In the case $\sigma_2 = 0$ equation (2.6) is satisfied and equation (2.5) serves to find the real decrement σ_1 as a function of all parameters. The boundary of stability is determined from the condition $\sigma_1 = 0$, i.e. $D = 0$, or

$$(1+\alpha)R + (1 + a + a/\alpha)R_d = 1 \qquad (2.7)$$

Each point in the plane RR_d corresponds to a state of equilibrium with given values of gradients of temperature and concentration A_0 and B_0. The Rayleigh numbers R and R_d may be either positive or negative, depending on the direction of the gradients. Equation (2.7) describes a straight line in the plane RR_d, which separates the region where $\sigma_1 < 0$ (attenuation of monotonic disturbances), from the region, where $\sigma_1 > 0$ (amplification of monotonic disturbances).

By analogous means we shall find the "neutral" line for periodic disturbances. For these disturbances $\sigma_2 \neq 0$ and the square bracket in (2.6) has to be equated to zero. Together with (2.5) this yields the system from which the decrement of disturbances σ_1 and their frequency σ_2 are determined. Upon eliminating σ_2 from the two equations we find the equation for the decrements of the periodic disturbances σ_1. On the boundary of stability $\sigma_1 = 0$. This gives $AD - BC = 0$; from this we find the equation of the "neutral" line for periodic disturbances

$$\begin{aligned} P_d[PP_d + (1+a)P_d - \alpha P]R + P[P + PP_d - (a/\alpha)P_d]R_d = \\ = [P + (1+a)P_d][(1+P)(1+P_d) + aP_d] \qquad (2.8) \end{aligned}$$

In the plane RR_d the lines (2.7) and (2.8) intersect each other at the point with the coordinates

$$R_* = -\frac{(a/\alpha)[1 + P + (1+a)P_d] + [(1+a)(1 + P_d + aP_d) + aP]}{P(1+\alpha) - P_d(1 + a + a/\alpha)} \tag{2.9}$$

$$R_{d*} = \frac{[1 + P + aP_d] + \alpha[1 + P + (1+a)P_d]}{P(1+\alpha) - P_d(1 + a + a/\alpha)} \tag{2.10}$$

We shall also find the equation which defines the frequency of the "neutral" oscillations

$$\sigma_2^2 = \frac{P(1+\alpha) - P_d(1 + a + a/\alpha)}{P_d[PP_d + (1+a)P_d - \alpha P]}(R_d - R_{d*}) \tag{2.11}$$

From formula (2.11) it is seen that the frequency of neutral oscillations becomes zero at the point of intersection of lines (2.7) and (2.8) and is real on the line (2.8) only on one side of this point. The line (2.8), therefore, has the meaning of a neutral line for periodic disturbances only on one side of the point of intersection, namely the point where $\sigma_2^2 > 0$. (This is in complete analogy to the case of convectional stability in a magnetic field [5] and in the case of rotation [6].)

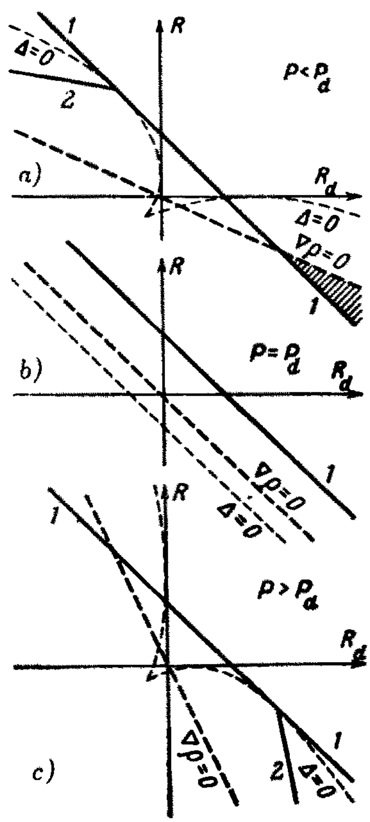

Fig. 1.

Equations (2.7) to (2.11) contain many parameters. In order to clarify the characteristic peculiarities of the situation we shall discuss below certain cases which are of greatest interest.

3. Consider first the simplest case for which the effects of thermal diffusion and of diffusive heat conductivity ($\lambda = 0$) may be neglected. In this case the basic equations (1.2) and the expressions for the fluxes (1.3) are significantly simplified and the new effects related to diffusion, which were mentioned in the introduction are particularly distinct. Equations of neutral curves for monotonic and vibrational disturbances may now be written in the following way (in equations (2.7) and (2.8) it must be assumed that $\alpha = a = 0$):

$$R + R_d = 1 \tag{3.1}$$

$$P_d^2 (1 + P) R + P^2 (1 + P_d) R_d = \\ = (1 + P)(1 + P_d)(P + P_d) \tag{3.2}$$

The coordinates of the point of intersection of lines (3.1) and (3.2) in the plane RR_d are

$$R_* = -\frac{1 + P_d}{P - P_d}, \qquad R_{d*} = \frac{1 + P}{P - P_d} \tag{3.3}$$

The critical frequency of the periodic disturbances on the line (3.2) is determined by the relationship

$$\sigma_2^2 = \frac{1}{P^2}\left(\frac{R}{R_*} - 1\right) \tag{3.4}$$

The distribution of the lines of stability in the plane RR_d is determined by the relation $P/P_d = D/\chi$. In Fig. 1 *a*, *b* and *c*, the curves of stability, are represented for the cases $P < P_d$, $P = P_d$ and $P > P_d$, respectively. In Fig. 1, line *1* is the neutral line for monotonic disturbances. The intersections of line *1* with the R-axis and R_d-axis, represent the critical Rayleigh numbers for the cases of purely thermal and purely concentrational instability. Line 2 in Fig. 1*a* and 1*b* represents the neutral line for periodic disturbances. The region of stability is situated under the lines *1* and 2. The curve $\Delta = 0$ is also represented in the figures, where Δ is the discriminant of the cubic equation (2.4)*. In the region below this curve the cubic equation (2.4) has two-complex conjugated roots which describe the periodic disturbances. The decrement of periodic disturbances becomes zero on line 2; in the region between the line 2 and the curve $\Delta = 0$ the periodic disturbances are amplified. Thus, in a mixture, in distinction to a pure medium, vibrational instability is possible. The only exception is the case $P = P_d$, i.e. $\chi = D$, when the point of intersection of lines *1* and 2 approaches infinity (Fig. 1*b*). In that case periodic disturbances are always damped and there exists only the monotonic instability in the region above line *1*.

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* **The extremum on the discriminant curve $\Delta = 0$ has the following coordinates**

$$R = \frac{P_d}{27P(P - P_d)}\left[2 - \frac{P}{P_d}(1 + P_d)\right]^3, \quad R_d = \frac{P}{27P_d(P_d - P)}\left[2 - \frac{P_d}{P}(1 + P)\right]^3$$

Contact with the axes R and R_d correspondingly occurs at the points

$$R = \frac{(P - P_d)(1 - P_d)}{P_d^2} \qquad R_d = \frac{(P_d - P)(1 - P)}{P^2}$$

In a pure medium, as is well known, instability arises if the density gradient is directed upward (heavier fluid on top) and if its magnitude exceeds a certain critical value. It might have been expected that also in a mixture the appearance of instability is determined in the last resort by the density gradient.

It turns out, however, that this is not so: the instability arises under certain conditions if the density of the mixture is everywhere the same and even if the gradient of density is directed downward.

It is easy to find a line in the RR_d plane, of which all the points represent a state of equilibrium of a mixture with zero density gradient.

The equation of this line, as seen from (1.1), is

$$P_d R + P R_d = 0 \tag{3.5}$$

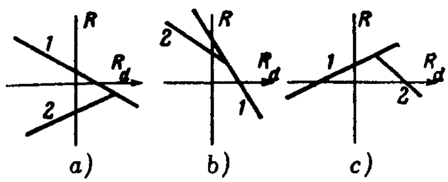
To the points in the RR_d plane, situated below the line (3.5), there correspond states of equilibrium with density gradient directed downward; in the region above the line (3.5) the gradient $\nabla\rho$ is directed upward. The line (3.5) in all cases, with the exception of $P = P_d$, intersects the lines *1* and *2* (Fig. 1). In each of the cases $P < P_d$ and $P > P_d$ there are, therefore, two regions for which the stability curve is located below the line $\nabla\rho = 0$ (one of such regions is shaded in Fig. 1*a*). All states of equilibrium, represented by the points inside these regions, are unstable, although for those states the medium on the bottom is heavier.

This somewhat unexpected result of the calculation may be visualized.

Consider, for example, the shaded region in Fig. 1*a*, where according to the calculation instability of the monotonic type occurs with the density gradient directed downward. In this region we have the temperature gradient, directed upward ($R < 0$; heat applied on top) and the concentration gradient directed downward ($R_d > 0$: lighter component dominant on the bottom). Here the inequality $P_d > P$ is also valid (i.e. $\chi > D$; equalization of temperature takes place faster than equalization of concentration).

Under these conditions an element of the medium which accidentally moved upward will be heated quickly, but it will lose its light component relatively slowly. In its new location the temperature of that element will, of course, be lower than the temperature of the neighboring medium, but it will be richer of the lighter component; if the gradients have suitable values, the density of the displaced element may become smaller than the density of the surrounding medium and, consequently, the element will continue to rise; instability of the monotonic type will occur. In a similar manner the appearance of periodic instability may be understood for the case when the medium on the bottom is heavier.

4. Results of Section 3 referred to the case $\lambda = 0$ (i.e. $\alpha = 0$, $a = 0$), when transport effects, namely the thermal diffusion and the diffusive heat conductivity may be neglected. If these effects are significant, the general equations (2.7) to (2.11) should be used. The situation described in Section 3, in general, is valid also when thermal diffusion and diffusive heat conductivity are taken into account. However, the relative configuration of the neutral lines of the monotonic and the periodic disturbances will now be determined by the four parameters of the medium (P, P_d, α, a) and it may be different from that represented in Fig. 1. From the general equations (2.7) to (2.11) we may easily obtain a classification of all the possible cases of relative configuration of the stability lines in the RR_d plane, depending on the relationships between the parameters. It is not necessary to present this classification here, inasmuch as some of the formally possible cases may be of little practical interest, because, for example, mixtures with an anomalous thermal diffusion ($\lambda > 0$) seldom occur, and the effect of diffusive heat conductivity is always very small. In Fig. 2 examples of the spectra of instability for certain values of the parameters are given. (The point $R = R_d = 0$ lies always in the stable region.) In connection with these examples it is noted that in the presence of thermal diffusion, a mixture (with respect to stability) differs from a pure medium, even if in that mixture a gradient of concentration is absent ($R_d = 0$). Consequently, as seen from Fig. 2*a*, instability for $R_d = 0$ occurs with sufficient heating on the bottom and below. It is seen further from Fig. 2*b* that when heating below for $R_d = 0$ instability occurs relative to the periodic disturbances, etc.



a) $a = 0, \alpha > \frac{P_d}{P}(1+P)$

b) $a = 0, \quad -1 < \alpha < -\frac{1+P}{1+P+P_d}$

c) $-\frac{a}{1+a} < \alpha < 0$

Fig. 2.

5. It is interesting to investigate the stability of the equilibrium for cases for which the equilibrium gradients of temperature and concentration are associated through a given relationship and therefore no longer independent, as was assumed above.

If, for example, the vertical canal is closed at top and bottom, so that the substance cannot go through, then for static equilibrium the mass flux is zero: $\mathbf{j} = 0$; here the heat flux $\mathbf{q}$ differs from zero and it is defined by the conditions of heating.

Another limiting case may be investigated, namely when $\mathbf{q} = 0$, but when mass flux occurs along the canal.

In both cases the gradients of temperature and of concentration differ from zero, they are connected, however, by the conditions $\mathbf{j} = 0$ or $\mathbf{q} = 0$.

Consider first the case $\mathbf{j} = 0$. Here the mass diffusive and thermal diffusive fluxes compensate each other and the gradients of temperature and of concentration are connected by the relation $\lambda A_0 + B_0 = 0$, as seen from the second equation of (1.3) and from (1.5).

In nondimensional form these functions are

$$-\alpha P_d R + P R_d = 0 \tag{5.1}$$

From equations (2.7) and (2.8), taking into account (5.1), we find expressions for the critical Rayleigh numbers R_1 and R_2 for the cases of monotonic and periodic disturbances respectively

$$R_1 = \frac{P}{(P + aP_d) + \alpha (P + P_d + aP_d)} \tag{5.2}$$

$$R_2 = \frac{[P + (1 + a) P_d] [(1 + P) (1 + P_d) + aP_d]}{P_d^2 [1 + (1 + \alpha) P]} \tag{5.3}$$

Figure 3 represents critical numbers R_1 and R_2 depending on the parameter of thermal diffusion α for constants P, P_d and a. Curves $R_1(\alpha)$ and $R_2(\alpha)$ intersect at $\alpha = \alpha_0$, where

$$\alpha_0 = -\frac{(P + aP_d)(1 + P + aP_d) + aP_d^2}{(P + P_d + aP_d)(1 + P + aP_d) + (1 + a) P_d^2} \tag{5.4}$$

This frequency of the neutral vibrations on the line $R_2(\alpha)$ equals

$$\sigma_2^2 = \frac{(P + P_d + aP_d)(1 + P + aP_d)}{PP_d^2 (1 + P + \alpha P)} (\alpha_0 - \alpha) \tag{5.5}$$

As seen from Fig. 3, in case of normal thermal diffusion ($\lambda < 0$, $\alpha > 0$) instability is possible only with respect to monotonic disturbances for which the critical number R_1 decreases with an increase of α. In the region of significant thermal diffusion, periodic instability is possible

$$-\left(1 + \frac{1}{P}\right) < \alpha < \alpha_0 \quad \text{branch } R_2$$

and likewise instability of the monotonic type when heating occurs on top

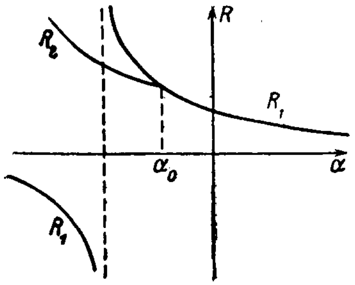


Fig. 3.

$$\alpha < -\frac{P + aP_d}{P + P_d + aP_d} \quad \text{lower branch of } R_1$$

Analogous relations exist for the case $\eta = 0$. The connection between R and R_d here is as follows:

$$-\alpha\,(1 + a)\,P_d R + aPR_d = 0 \qquad (5.6)$$

We shall give only the critical values of the diffusive Rayleigh number for the monotonic and the periodic disturbances

$$R_{d1} = \frac{(1 + a)\,P_d}{[(1 + a)^2 P_d + aP] + (a\,/\,\alpha)\,[P + (1 + a)\,P_d]} \qquad (5.7)$$

$$R_{d2} = \frac{(1 + a)(P + P_d + aP_d)[(1 + P)(1 + P_d) + aP_d]}{P^2\,[1 + (1 + a + a\,/\,\alpha)\,P_d]} \qquad (5.8)$$

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